Eur. Phys. J. D **23**, 109–115 (2003)
DOI: 10.1140/epjd/e2003-00024-x

THE EUROPEAN PHYSICAL JOURNAL D

Electroweak interactions between intense neutrino beams and dense electron-positron magnetoplasmas

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Received 11 September 2002
Published online 4 February 2003 –

Abstract. The electroweak coupling between intense neutrino beams and strongly degenerate relativistic dense electron-positron magnetoplasmas is considered. The intense neutrino bursts interact with the plasma due to the weak Fermi interaction force, and their dynamics is governed by a kinetic equation. Our objective here is to develop a kinetic equation for a degenerate neutrino gas and to use that equation to derive relativistic magnetohydrodynamic equations. The latter are useful for studying numerous collective processes when intense neutrino beams nonlinearly interact with degenerate, relativistic, dense electron-positron plasmas in strong magnetic fields. If the number densities of the plasma particles are of the order of 10^{33} cm^{-3}, the pair plasma becomes ultra-relativistic, which strongly affects the potential energy of the weak Fermi interaction. The new system of equations allows several neutrino-driven streaming instabilities involving new types of relativistic Alfvén-like waves. The relevance of our investigation to the early universe and supernova explosions is discussed.

PACS. 95.30.Qd Magnetohydrodynamics and plasmas – 97.60.Bw Supernovae – 97.10.Cv Stellar structure, interiors, evolution, nucleosynthesis, ages

1 Introduction

The neutrino was introduced by Wolfgang Pauli in 1930 to explain a problem concerning nuclear beta decay in which a neutral particle is emitted together with an electron so that the sum of the energies of the neutral particle and the electron is almost constant. Enrico Fermi in 1933 formulated a theory of beta decay. This theory involved a new interaction in which the neutron changed into a proton, an electron and an antineutrino. It is well-known [1,2] that neutrinos were copiously produced in the early universe, as well as in supernova explosions and in the sun. Neutrinos are the most slippery and elusive of the subatomic particles. They are created in connection with numerous plasma processes [1–6] and they are supposed to carry almost no mass and no electric charge in vacuum. However, when neutrinos propagate through the plasma, their propagation characteristics are significantly modified due to their interactions with the plasma electrons and the W^+ bosons. Specifically, the energy spectrum of the neutrinos acquires contributions due to the electroweak Fermi-interactions [7–9]. The latter are associated with a force which produces charged currents due to the neutrino-plasma couplings.

Recently, there has been a great deal of interest [10–18] in investigating numerous collective processes in neutrino plasma physics, and to transfer knowledge from plasma physics to elementary particle physics, high energy astrophysics, and cosmology. The most important stimulus to the development of neutrino plasma physics has been associated with the understanding of several outstanding puzzles in astrophysical objects and also in the early universe. For example, recent theoretical works [13,15–17] suggest that intense neutrino beams interact with a dense plasma in a collective manner due to the electro-weak Fermi interaction. This nonlinear neutrino-plasma coupling turns out to be significant because the potential energy [8] of the Fermi-interaction is proportional to the number densities of the plasma particles. Nonlinear effects produced by the neutrino driving force [10,11] include the generation of plasma waves [12,13,15], density inhomogeneities [17], and magnetic fields [18]. Studies of nonlinear neutrino-plasma interactions are of significant importance in connection with anomalous absorption of neutrinos causing

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supernova explosions, as well as with the generation of magnetic fields on the surfaces of neutron stars.

A supernova explosion occurs when the inner part of a star is very dense (mass density $\rho \sim 10^{14}$ g/cm^3) and in a very short time (about 1 millisecond). In such a dense medium, the electrons and positrons as well as the neutrinos are in a strongly degenerate state [19–21]. In this case, we have an ultra-relativistic Fermi gas. Furthermore, the magnetic field generated during the supernova explosions may approach the Schwinger's value $B \sim m_0^2 c^3/e \simeq 4.4 \times 10^{13}$ G and more, where m_0 is the electron rest mass, c is the speed of light in vacuum, and e is the magnitude of the electron charge. As was shown by Bisnovatyi-Kogan [22], the presence of rotation may increase the magnetic field by an additional factor of 10^3–10^4. Obviously, in the presence of such magnetic fields it is necessary to consider the thermodynamics of the electron-positron and neutrino medium, the quantization of the energy of the particles, etc.

Influence of an external magnetic field on the propagation of neutrinos in an electron plasma was examined in references [23–25]. This result was applied by Kusenko and Segrè [26] to explain the birth (kick) velocity of newly born pulsars. Shukla and Stenflo [27] investigated the propagation of neutrinos in an electron plasma with an equilibrium density gradient and a sheared magnetic field. Very recently, Kuznetsov and Mikheev [28] considered neutrino interactions with a strongly magnetized electron-positron plasma. They carried out studies of the average losses of the neutrino energy and momentum as well as of the integral action of the neutrinos. Gvozdev and Ognev [29] estimated the efficiency of electron-positron pair production by the neutrino flux from the accretion disk of a Kerr black hole.

In the present paper, we consider the electrodynamics of an electron-positron-neutrino gas in the presence of a strong magnetic field. Namely, we consider the electron-positron dense plasma, when collisions between particles are very frequent. In this case, the mean velocities of the different components of the gas must, in fact, be almost equal. Based on this fact, we develop relativistic magnetohydrodynamic (RMHD) equations for the nonlinearly interacting neutrinos and the electron-positron pairs in a strong external magnetic field. In our consideration, the conservation of the number of all particles is satisfied, *i.e.*, the neutrino scattering processes on electron-positrons, $\nu e^{\mp} \to \nu e^{\mp}$, take place. The newly derived RMHD equations are then used to consider the propagation of Alfvén-like and MHD waves in the presence of neutrino beams. The dispersion relations for different interesting cases are obtained and analyzed. Some new wave modes and instabilities are found. The relevance of our investigation to MHD turbulence in astrophysical settings is also discussed.

2 Interaction energies

In relativistic quantum theory the electron (positron) energy levels $\varepsilon_{j,\delta}$ in a strong magnetic field B, including Pauli paramagnetism as well as Landau diamagnetism, can be expressed as [30]

$$\varepsilon_{j,\delta} = c\sqrt{p_z^2 + \frac{e\hbar B}{2c}(2j+1+\delta) + m_0^2c^2}, \tag{1}$$

where p_z is the momentum component along the magnetic field direction, m_0 is the rest mass of the electrons (positrons) and $\delta = \pm 1$; the latter defines the direction of the spin of the particles in the magnetic field. Furthermore, $2\pi\hbar$ is the Planck constant, and $j = 0, 1, 2...$

Here we consider the case of a strong magnetic field satisfying $\sqrt{eB\hbar c} - \mu \gg T$, where μ is the chemical potential, so that only the Landau ground level is filled ($j = 0$), *i.e.* we are taking into account only the Pauli paramagnetism and the self-energy of the particles. Thus

$$\varepsilon_{0,\delta} = c\sqrt{p_z^2 + \frac{e\hbar B}{2c}(1+\delta) + m_0^2c^2}. \tag{2}$$

We know that the effective electroweak interaction (scalar potential) energy of an electron-positron gas with neutrinos in the steady state is

$$U_{ep} = \sqrt{2}G_{\mathrm{F}}\left(\frac{1}{2} + 2\sin^2\Theta_{\mathrm{W}}\right)(n_e - n_p), \tag{3}$$

where $G_{\mathrm{F}} = 9 \times 10^{-38}$ eV cm^3 is the Fermi constant for weak nuclear interactions, $\sin^2\Theta_{\mathrm{W}} \sim 0.23$, and n_e and n_p are the number densities of the electrons and positrons, respectively. The contributions of neutrons and anti-neutrons to equation (3) have been neglected because we consider the case where their number densities are much smaller than the electron and positron number densities. We are also discarding the contributions of the anisotropies of the electron/positron and neutrino distribution functions [11] as well as of the effective electroweak vector potential coupling between the pairs and the neutrinos [14]. This is justified as long as $\mathbf{v}_{e,p}\cdot\mathbf{v}_\nu \ll c^2$, where $\mathbf{v}_{e,p}$ is the mean velocity of the electrons/positrons and $\mathbf{v}_\nu$ is the neutrino velocity. For a non-stationary scenario, the total force $\mathbf{f}_{ep}$ on the neutrinos is [14] approximately

$$\mathbf{f}_{ep} = -\sqrt{2}G_{\mathrm{F}}\left[\boldsymbol{\nabla}(n_e - n_p) + c^{-2}\partial_t(\mathbf{J}_e - \mathbf{J}_p) - c^{-2}\mathbf{v}_\nu \times \nabla \times (\mathbf{J}_e - \mathbf{J}_p)\right],$$

where $\mathbf{J}_j = n_j\mathbf{v}_j$.

Taking into account the fact that in a strong magnetic field the number J of energy levels per unit volume in an interval Δp_z is

$$J = \frac{eB\Delta p_z}{4\pi^2\hbar^2 c}, \tag{4}$$

we rewrite the expression (3) as

$$U_{ep} = \sqrt{2}G_{\mathrm{F}}\frac{eB}{c}\frac{1}{2\pi^2\hbar^2}\int \mathrm{d}p_z\left(\frac{1}{\mathrm{e}^{\frac{\varepsilon-\mu}{T}}+1} - \frac{1}{\mathrm{e}^{\frac{\varepsilon+\mu}{T}}+1}\right), \tag{5}$$

where $\varepsilon = \epsilon_{0,\delta}$. In the strongly degenerate case, we obtain

$$U_{ep} = \sqrt{2}G_{\mathrm{F}}\frac{eB}{\hbar c}\frac{\left(3\pi^2\right)^{1/3}}{2\pi^2}n^{1/3}, \tag{6}$$

where n is a density, defined in terms of the Fermi momentum $p_{\rm F}$, *i.e.* $n = (p_{\rm F}/\hbar)^3/3\pi^2$.

But if the electron-positron gas influences the neutrino behavior, the reverse can also exist. The neutrinos per unit volume can act on the electrons (positrons) in a potential field, which is proportional to the density of the neutrinos n_ν, *i.e.* the neutrino gas interacts with each electron and positron through a potential energy

$$U_\nu = \sqrt{2} G_{\rm F} n_\nu. \tag{7}$$

We note that according to references [6,10], we have neglected non-potential interactions in (7). In general, the neutrino driving force acting on the electrons is of the form [14] $\mathbf{f}_\nu = -\sqrt{2}G_{\rm F}\left(\boldsymbol{\nabla} n_\nu + c^{-2}\partial_t \mathbf{J}_\nu\right)$, where $\mathbf{J}_\nu = n_\nu \mathbf{v}_\nu$ is determined from $\partial_t n_\nu + \boldsymbol{\nabla}\cdot\mathbf{J}_\nu = 0$. It turns out that the $\partial_t \mathbf{J}_\nu$ contribution is insignificant as long as the phase speed of the plasma disturbance is much smaller than the speed of light.

3 Relativistic hydrodynamic equations for the neutrino gas

For a neutrino gas in an isotropic plasma one can ignore the spin of the neutrinos and derive a kinetic equation for the distribution function (spectral function) of the neutrinos $N_\nu(\mathbf{k},\omega_\nu,\mathbf{r},t)$ or $N_\nu(\mathbf{p}_\nu,\varepsilon_\nu,\mathbf{r},t)$, where $\mathbf{p}_\nu = \hbar\mathbf{k}$ and $\varepsilon_\nu = \hbar\omega_\nu$, *i.e.*

$$\frac{\partial N_\nu}{\partial t} + \frac{c^2}{\varepsilon_\nu}(\mathbf{p}_\nu\cdot\boldsymbol{\nabla})N_\nu + \mathbf{f}_{ep}\cdot\frac{\partial N_\nu}{\partial \mathbf{p}_\nu} = 0. \tag{8}$$

Using the results of the previous section, we have expressions for the forces $\mathbf{f}_{ep} = -\boldsymbol{\nabla}U_{ep} - \sqrt{2}G_{\rm F}c^{-2}\partial_t(\mathbf{J}_e - \mathbf{J}_p)$, where U_{ep} is the Fermi weak interaction potential energy, given by the expressions (3, 5, 6).

We now start from equation (8) to obtain a set of relativistic fluid equations [10] for the macroscopic parameters of the neutrino gas (the neutrino density n_ν, the temperature T_ν, the entropy, and the average velocity $\mathbf{u}_\nu$). We already noted in Section 1 that to have a mutual influence of neutrinos and electrons (positrons) on each other, it is necessary to have large densities of each species of particles. But in this case, one has a strong degenerate Fermi gas or relativistic temperatures. In both cases, the mean value of the kinetic energy and the mean density of the particles have to be obtained in their proper reference frame. Also, the mass of the particles depends on the density (strongly degenerate gas) or on the temperature in a complex way.

We first introduce our basic definitions in the rest system, namely the neutrino density and the mean velocity

$$n'_\nu(\mathbf{r},t) = \int \mathrm{d}\mathbf{p}'_\nu N'_\nu, \tag{9}$$

and

$$\mathbf{u}_\nu = \frac{c^2}{n'_\nu}\int \frac{\mathrm{d}\mathbf{p}'_\nu}{\varepsilon'_\nu}\mathbf{p}'_\nu N'_\nu, \tag{10}$$

where the prime denotes the quantities in the rest system.

The transition to the laboratory reference frame can be realized with the help of the Lorentz transformation for the energy and momentum of the neutrino

$$\varepsilon_\nu = \gamma_\nu(\varepsilon'_\nu + \mathbf{u}_\nu\cdot\mathbf{p}'_\nu), \qquad p_{i\nu} = s^\nu_{ik}p'_{k\nu} + c^{-2}\gamma_\nu u_{i\nu}\varepsilon'_\nu,$$
$$s^\nu_{ij} = \delta_{ij} + (\gamma_\nu - 1)\frac{u_{i\nu}u_{j\nu}}{u_\nu^2} \text{ and } \gamma_\nu = (1 - u_\nu^2/c^2)^{-1/2}. \tag{11}$$

In the relativistic system, there are several kinds of invariant quantities. For example, the distribution function of the particles and the ratio $\mathrm{d}\mathbf{p}/\varepsilon$ are invariant quantities. Hence

$$\frac{\mathrm{d}\mathbf{p}_\nu}{\varepsilon_\nu} = \frac{\mathrm{d}\mathbf{p}'_\nu}{\varepsilon'_\nu}\cdot \tag{12}$$

Using the Lorentz transformation for the energy and the momentum of neutrinos, we obtain from equation (8) the equation of continuity

$$\frac{\partial n_\nu}{\partial t} + \boldsymbol{\nabla}\cdot(n_\nu\mathbf{u}_\nu) = 0, \tag{13}$$

where $n_\nu = n'_\nu\gamma_\nu$ and

$$\mathbf{u}_\nu = \frac{c^2}{n_\nu}\int \mathrm{d}\mathbf{p}_\nu\frac{\mathbf{p}_\nu}{\varepsilon_\nu}N_\nu. \tag{14}$$

The momentum conservation equation is

$$\frac{1}{c^2}\left(\frac{\partial}{\partial t} + \mathbf{u}_\nu\cdot\boldsymbol{\nabla}\right)(\gamma_\nu W'_\nu\mathbf{u}_\nu) = -\frac{1}{n_\nu}\boldsymbol{\nabla}P_\nu + \mathbf{f}_{ep}, \tag{15}$$

and for the energy of the neutrinos we have

$$\frac{\partial}{\partial t}\left(\gamma_\nu n_\nu\langle\varepsilon'_\nu\rangle + \gamma_\nu^2\frac{u_\nu^2}{c^2}P_\nu\right) + \boldsymbol{\nabla}\cdot(\gamma_\nu n_\nu W'_\nu\mathbf{u}_\nu) = n_\nu\mathbf{u}_\nu\cdot\mathbf{f}_{ep}, \tag{16}$$

where $\langle\varepsilon'_\nu\rangle$ is the internal energy of neutrinos $\langle\varepsilon'_\nu\rangle = (1/n_\nu)\int\mathrm{d}\mathbf{p}'_\nu N'_\nu\varepsilon'_\nu$, P_ν is the pressure of the neutrinos, and the enthalpy W'_ν (a heat function) of the neutrino is

$$W'_\nu = \langle\varepsilon'_\nu\rangle + \frac{\gamma_\nu P'_\nu}{n_\nu} = \langle\varepsilon'_\nu\rangle + \frac{P'_\nu}{n'_\nu}\cdot \tag{17}$$

The last term in the right-hand side of equations (15, 16) describes the electro-weak coupling between the neutrinos and pairs.

Equations (13–16) are valid for arbitrary temperatures and densities. Furthermore, we can now consider interactions between neutrino and electron-positron gases when they are strongly degenerate and relativistic. Also we note that for densities of particles up to 10^{32} cm^{-3}, the Fermi momentum $p_{\rm F}$ is much larger than $m_\alpha c$ ($\alpha = e,p,\nu$), so that both the neutrinos and the pairs become ultra-relativistic. In this case, we can write for the internal energy of the particles, $\langle\varepsilon'_\alpha\rangle = (3/4)\varepsilon_{\rm F\alpha}$, for the pressure $P'_\alpha = n'_\alpha\varepsilon_{\rm F\alpha}/4$, for the enthalpy $n'_\alpha W'_\alpha = n'_\alpha\langle\varepsilon'_\alpha\rangle + P'_\alpha$, where $\varepsilon_{\rm F\alpha} = (3\pi^2)^{1/3}\hbar c n'^{1/3}_\alpha$. For convenience, we will introduce the effective mass of the particle as $m_\alpha = \varepsilon_{\rm F\alpha}/c^2 = (3\pi^2)^{1/3}(\hbar/c)n'^{1/3}_\alpha$.

4 MHD equations for the pairs

We now present a set of relativistic hydrodynamic equations for the electron-positron fluid [31], including the neutrino driving force. We have

$$\frac{\partial n}{\partial t} + \boldsymbol{\nabla} \cdot (n\mathbf{u}) = 0, \tag{18}$$

$$\frac{1}{c^2}\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\boldsymbol{\nabla}\right)(\gamma W \mathbf{u}) = -\frac{1}{n}\boldsymbol{\nabla} P + \mathbf{f}, \tag{19}$$

and

$$\frac{\partial}{\partial t}\left(\gamma n\left\langle\varepsilon\right\rangle + \gamma^2\frac{u^2}{c^2}P + U\right) + \boldsymbol{\nabla}\cdot(\gamma n W\mathbf{u} + \mathbf{S}) = -2\sqrt{2}G_{\mathrm{F}}n(\mathbf{u}\cdot\boldsymbol{\nabla})n_\nu, \tag{20}$$

where $n = \gamma n'$ is the density of the electrons (positrons) in the laboratory system $\gamma = \left(1 - u^2/c^2\right)^{-1/2}$ is the relativistic gamma factor for the electrons (positrons), $\mathbf{u}$ is the mean velocity of the $e^- + e^+$ fluids, $W = \langle\varepsilon\rangle + \gamma P/n$ is the heat function (enthalpy), $\langle\varepsilon\rangle$ and $P = nT$ are the interval energy and pressure, respectively, $\mathbf{S} = (c/4\pi)\mathbf{E}\times\mathbf{B}$ is the Poynting vector, $U = (E^2 + B^2)/8\pi$ is the electromagnetic energy density, and $\mathbf{f}$ is the force which acts on the $e^- + e^+$ fluid. The components of $\mathbf{f}$ are

$$f_i = -\frac{1}{n}\frac{\partial T_{ij}}{\partial x_j} - \frac{1}{nc^2}\frac{\partial S_i}{\partial t} - 2\sqrt{2}G_{\mathrm{F}}\left(\nabla_i n_\nu + \frac{1}{c^2}\frac{\partial \mathbf{J}_{\nu i}}{\partial t}\right), \tag{21}$$

where $T_{ij} = (1/4\pi)\left[-E_iE_j - B_iB_j + (1/2)\delta_{ij}(E^2 + B^2)\right]$ is the flux tensor of the electromagnetic field energy density. The last terms in the right-hand side of equations (20, 21) represent the contributions associated with the neutrino driving force including the time derivative neutrino flux $\mathbf{J}_\nu$ that is determined from $\partial_t n_\nu + \boldsymbol{\nabla}\cdot\mathbf{J}_\nu = 0$.

In the case of an ideal conducting medium, the inductive electric field satisfies the relation $\mathbf{E} = -c^{-1}\mathbf{u}\times\mathbf{B}$, and for the magnetic field we have the induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla}\times(\mathbf{u}\times\mathbf{B}). \tag{22}$$

5 Neutrino driven MHD waves

Let us consider the propagation of small perturbations in a homogeneous magnetoactive plasma-neutrino beam. We then linearize (13–16) with respect to the perturbations, which are represented as $n_\nu = n_{0\nu} + \delta n_\nu$ and $\mathbf{u}_\nu = \mathbf{u}_{0\nu} + \delta\mathbf{u}_\nu$, where the subscript 0 denotes the constant equilibrium value, and δn_ν and $\delta\mathbf{u}_\nu$ represent small variations. After linearization of (13–16), we look for plane wave solutions that are proportional to $\exp[\mathrm{i}(\mathbf{k}\cdot\mathbf{r} - \omega t)]$, where $\mathbf{k}$ and ω are the wave vector and the frequency, respectively.

To derive the dispersion relation, we should remember that (15, 16) as well as (19, 20) contain the internal energy, enthalpy and pressure, which are defined in the rest system. Linearizing them and writing them in the laboratory system, we obtain from (17)

$$\delta W = \delta\varepsilon_{\mathrm{F}} = \frac{\varepsilon'_{0\mathrm{F}}}{3}\frac{\delta n'}{n'_0} = \frac{\varepsilon_{0\mathrm{F}}}{3}\left(\frac{\delta n}{n_0} - \gamma_0^2\frac{u_0}{c^2}\delta u_z\right), \tag{23}$$

which coincides with $\delta P'/n'$. Furthermore, the perpendicular and parallel components of the particle momentum are given by, respectively,

$$\delta\mathbf{p}_\perp = m_0\gamma_0\delta\mathbf{u}_\perp$$

and

$$\delta p_z = \frac{m_0\gamma_0}{3}\left[(2\gamma_0^2 + 1)\delta u_z + u_0\frac{\delta n}{n_0}\right], \tag{24}$$

where $m_0 = W_0/c^2 = \varepsilon_{0F}/c^2$ and $u_0 = u_{0z}$. The perturbations of the Poynting vectors are $\delta S_\parallel = 0$ and

$$\delta\mathbf{S}_\perp = \frac{B_0^2}{4\pi}\left(\delta\mathbf{u}_\perp - u_0\frac{\delta\mathbf{B}_\perp}{B_0}\right), \tag{25}$$

and for the perturbation of T_{ij} we have

$$\delta T_{ij} = \frac{1}{4\pi}\left(\delta_{ij}B_0\delta B_z - B_{0i}\delta B_j - B_{0j}\delta B_i\right). \tag{26}$$

Using the relations (23, 24) for the electron-positron fluid, we obtain a set of linear equations, describing the excitation of waves in an electron-positron fluid in the presence of a relativistic neutrino flux.

The linear continuity equation for the electron-positron fluid is

$$(\omega - k_z u_0)\frac{\delta n}{n_0} = \mathbf{k}_\perp\cdot\delta\mathbf{u}_\perp + k_z\delta u_z. \tag{27}$$

The relation between the momentum and velocity were given by (17), so that

$$(\omega - k_z V)\frac{\delta n}{n_0} = \frac{\mathbf{k}_\perp\cdot\boldsymbol{\delta}\mathbf{p}_\perp}{m_0\gamma_0} + \frac{3k_z\delta p_z}{m_0\gamma_0(2\gamma_0^2 + 1)} \tag{28}$$

where $V = 2u_0\gamma_0^2/(2\gamma_0^2 + 1)$.

The equations of motion for $\delta\mathbf{p}_\perp$ and δp_z are

$$\left[\left(1 + \frac{V_A^2}{c^2}\right)\omega - k_z u_0\right](\mathbf{k}_\perp\cdot\delta\mathbf{p}_\perp) = \frac{m_0\gamma_0 c^2 k_\perp^2}{(2\gamma_0^2 + 1)}\frac{\delta n}{n_0} - \frac{u_0 k_\perp^2\delta p_z}{(2\gamma_0^2 + 1)} + \left(k^2V_A^2 - k_z u_0\omega\frac{V_A^2}{c^2}\right)m_0\gamma_0\frac{\delta B_z}{B_0} + \varepsilon_\nu k_\perp^2\frac{\delta n_\nu}{n_{0\nu}}, \tag{29}$$

and

$$(\omega - k_z V)\,\delta p_z = \frac{m_0\gamma_0 c^2}{(2\gamma_0^2 + 1)}k_z\frac{\delta n}{n_0} + \varepsilon_\nu k_z\frac{\delta n_\nu}{n_{0\nu}}, \tag{30}$$

with $V_A = B_0/(4\pi n_0 m_0)^{1/2}$ and $\varepsilon_\nu = 2\sqrt{2}G_{\mathrm{F}}n_{0\nu}(1 - \omega^2/k^2c^2)$. From equation (22) we obtain

$$(\omega - k_z u_0)\frac{\delta \mathbf{B}_\perp}{B_0} = -\frac{k_z \delta \mathbf{p}_\perp}{m_0\gamma_0}, \tag{31}$$

$$(\omega - k_z u_0)\frac{\delta B_z}{B_0} = \frac{\mathbf{k}_\perp \cdot \boldsymbol{\delta}\mathbf{p}_\perp}{m_0\gamma_0}, \tag{32}$$

and $\mathbf{k}_\perp \cdot \delta \mathbf{B}_\perp + k_z \delta B_z = 0$. To equations (28–32) we should add the equations for the neutrino flux in order to make the set of equations closed.

As for the electron-positron plasma, the neutrino gas is also in a degenerate state. We obtain for the density perturbation of the neutrinos

$$\left[(\omega - k_z V_\nu)^2 - \frac{c^2}{(2\gamma_{0\nu}^2+1)}\left(k_\perp^2 + \frac{3k_z^2}{(2\gamma_{0\nu}^2+1)}\right)\right]\frac{\delta n_\nu}{n_{0\nu}} = \left[k_\perp^2 + \frac{3k_z^2}{(2\gamma_{0\nu}^2+1)}\right]\frac{\delta U_{ep}}{m_{0\nu}\gamma_{0\nu}}, \tag{33}$$

where $V_\nu = 2u_{0\nu}\gamma_{0\nu}^2/(2\gamma_{0\nu}^2+1)$, and δU_{ep} is the perturbation of the potential energy (*e.g.* Eq. (6)) of the Fermi weak interaction, from which by using equation (24), we obtain for δU_{ep}

$$\delta U_{ep} = A\left[\frac{\gamma_0^2(\omega - k_z u_0)}{(2\gamma_0^2+1)(\omega - k_z V)}\frac{\delta n}{n_0} - \frac{k_z u_0 \gamma_0 \varepsilon_\nu}{m_0 c^2(2\gamma_0^2+1)(\omega - k_z V_\nu)}\frac{\delta n_\nu}{n_{0\nu}} + \frac{\delta B_z}{B_0}\right], \tag{34}$$

where $A = \sqrt{2}G_{\mathrm{F}}eB_0\varepsilon_{\mathrm{F}}/\pi^2\hbar^2c^2$.

If the mean velocity of the electron-positron fluid (gas) is zero, then for $\gamma_0 = 1$ we obtain from equation (34)

$$\delta U_{ep} = A\left(\frac{1}{3}\frac{\delta n}{n_0} + \frac{\delta B_z}{B_0}\right)\cdot \tag{35}$$

On the other hand, for $u_0 \sim c$ and $\gamma_0^2 \gg 1$, we have from equation (34)

$$\delta U_{ep} = A\left[\frac{1}{2}\frac{\delta n}{n_0} - \frac{k_z u_0}{2(\omega - k_z V_\nu)}\frac{\varepsilon_\nu}{m_0 c^2 \gamma_0}\frac{\delta n_\nu}{n_{0\nu}} + \frac{\delta B_z}{B_0}\right]\cdot \tag{36}$$

The equations which we obtained contain two relativistic fluxes, namely the electron-positron with the velocity u_0, and the neutrino with $u_{0\nu}$. For illustrative purposes, both fluxes are supposed to be directed along the z-axis, as the interaction is rather effective for this case. We note also the set of equations describing the excitation of relativistic Alfvén waves.

Let us now consider different types of instabilities.

First, we suppose that the electron-positron gas is incompressible, *i.e.* $\delta n = 0$. Here, δU_{ep} is given by

$$\delta U_{ep} = A\left[-\frac{k_z u_0 \gamma_0 \varepsilon_\nu}{m_0 c^2(2\gamma_0^2+1)(\omega - k_z V_\nu)}\frac{\delta n_\nu}{n_{0\nu}} + \frac{\delta B_z}{B_0}\right]\cdot \tag{37}$$

Let us consider the case $\mathbf{k}_\perp = 0$, which means $\delta B_z = 0$. Substituting (37) into (33) for the coinciding roots, *viz.* $\omega = k_z u_{0\nu} + \left[\sqrt{3}k_z c/(2\gamma_0^2+1)\right] + \mathrm{i}\delta$ and $\omega = k_z u_0 + \mathrm{i}\delta$, we obtain

$$\delta^2 = \frac{\sqrt{3}}{2}\frac{k_z^2}{\gamma_{0\nu}\gamma_0}\frac{A}{m_0}, \tag{38}$$

and for the growth rate we have

$$\delta \simeq 10^{-18}\sqrt{B_0}k_z c. \tag{39}$$

To deduce the above expression, we have taken $n_{0\nu} \sim 10^{36}$ cm^{-3}, $n_{0e} \approx n_{0p} \sim 10^{33}$ cm^{-3} and $\gamma_0 \sim 10$. For typical magnetic field strengths [23,24], *viz.* $B_0 \sim 10^{12}$ Gauss, and MeV neutrinos in the supernovae, we have

$$\delta \sim 10^8 \ \mathrm{s}^{-1}. \tag{40}$$

When $\mathbf{k}_\perp \neq \mathbf{0}$, the second term dominates in the expression (37), and from (30, 32, 28) at $\delta n = 0$, we obtain the equation for δB_z. The result is

$$\frac{\delta B_z}{B_0} = -\frac{3\varepsilon_\nu k_z^2}{m_0\gamma_0(2\gamma_0^2+1)}\frac{1}{(\omega - k_z u_0)^2}\frac{\delta n_\nu}{n_{0\nu}}\cdot \tag{41}$$

Substituting this expression into (37) and using equation (33), we obtain

$$\left[(\omega - k_z V_{0\nu})^2 - \frac{c^2}{(2\gamma_{0\nu}^2+1)}\left(k_\perp^2 + \frac{3k_z^2}{(2\gamma_{0\nu}^2+1)}\right)\right](\omega - k_z u_0)^2 + \left[k_\perp^2 + \frac{3k_z^2}{(2\gamma_{0\nu}^2+1)}\right]\frac{3c^2 A}{m_0\gamma_0\gamma_{0\nu}}\frac{k_z^2}{(2\gamma_0^2+1)} = 0. \tag{42}$$

Letting $\omega = k_z V_{0\nu} - [c/(\gamma_0^2+1)]\sqrt{k_\perp^2(2\gamma_{0\nu}^2+1) + 3k_z^2} + \mathrm{i}\delta$ and $\omega = k_z u_0 + \mathrm{i}\delta$ in equation (42) we obtain a cubic equation for δ, and the growth rate is

$$\delta \sim \frac{\sqrt{3}}{2}\left[\frac{3\sqrt{k_\perp^2(2\gamma_0^2+1) + 3k_z^2}cAk_z^2}{2m_0\gamma_0\gamma_{0\nu}(2\gamma_0^2+1)}\right]^{\frac{1}{3}}\cdot \tag{43}$$

Now we consider a more general case, namely when the density of the electron-positron plasma is compressible. In this case, from equations (28–32), we obtain the relation between δn and δn_ν including the expression for the perturbation of the magnetic field. The result is

$$\left[\left((\omega - k_z u_0)^2 - \frac{3k_z^2c^2}{(2\gamma_0^2+1)^2}\right)\left((\omega - k_z u_0)^2 - k^2V_A^2 + \omega^2\frac{V_A^2}{c^2}\right) - \frac{k_\perp^2 c^2(\omega - k_z u_0)^2}{2\gamma_0^2+1}\right]\frac{\delta n}{n_0} = \frac{\varepsilon_\nu}{m_0\gamma_0}\left[k_\perp^2(\omega - k_z u_0)^2 + \frac{3k_z^2}{(2\gamma_0^2+1)}\left((\omega - k_z u_0)^2 - k^2V_A^2 + \omega^2\frac{V_A^2}{c^2}\right)\right]\frac{\delta n_\nu}{n_{0\nu}}, \tag{44}$$

and

$$\left[(\omega - k_z u_0)^2 - k^2V_A^2 + \omega^2\frac{V_A^2}{c^2}\right]\frac{\delta B_z}{B_0} = \frac{k_\perp^2 c^2}{(2\gamma_0^2+1)}\frac{\delta n}{n_0} + \frac{k_\perp^2\varepsilon_\nu}{m_0\gamma_0}\frac{\delta n_\nu}{n_{0\nu}}\cdot \tag{45}$$

We consider first relativistic Alfvén waves at $u_0 = 0$, *i.e.* in (44, 45) we suppose that $\gamma_0 = 1$ and $V_A^2 \gg c^2$. Let us investigate the left-hand side in (44). In this case, we can rewrite (44) as

$$\left(\omega^2 - \omega_-^2\right)\left(\omega^2 - \omega_+^2\right)\frac{\delta n}{n_0} = \frac{\varepsilon_\nu c^2}{m_0 V_A^2} \times \left[k_\perp^2 \omega^2 + k_z^2 V_A^2\left(\frac{\omega^2}{c^2} - k^2\right)\right]\frac{\delta n_\nu}{n_{0\nu}}, \quad (46)$$

where $\omega_- = k_z c/\sqrt{3}$ is the frequency of the new sound waves, and $\omega_+ = kc$. This unusual dependence of the frequency on the angle is due to strong magnetic fields, which suppress the propagation of the usual sound waves perpendicular to the magnetic field direction. In deducing the left-hand side of equation (46), we have neglected a term $k_\perp^2 c^4 \omega^2/3V_A^2$ in view of the assumption $c^2 \ll V_A^2$.

First, we consider the excitation of relativistic Alfvén waves, which means that we let $\omega \sim \omega_+$ and $\omega \gg \omega_-$. These conditions allow us to simplify the expressions (34, 44, 46). Then the dispersion equation has the form

$$\left[(\omega - k_z V_\nu)^2 - q^2 c_\nu^2\right]\left(\omega^2 - \omega_+^2\right) = \frac{2q^2k^2}{3}\frac{Ac^2}{\gamma_{0\nu} m_0}, \quad (47)$$

where $c_\nu = c/\sqrt{2\gamma_{0\nu}^2 + 1}$ and $q^2 = k_\perp^2 + 3k_z^2/(2\gamma_{0\nu}^2 + 1)$. The growth rate in this case turns out to be

$$\delta \simeq \left(\frac{2qk}{3c_\nu}\frac{AcB_0}{\gamma_{0\nu} m_0}\right)^{\frac{1}{2}}. \quad (48)$$

For the same parameters, as given above, we have

$$\delta \sim (10^{-13}B_0)^{1/2}k. \quad (49)$$

Now we consider the excitation of the new waves by the neutrino flux. From equations (33, 44–46) we obtain

$$\left[(\omega - k_z V_\nu)^2 - q^2 c_\nu^2\right]\left(\omega^2 - \omega_-^2\right) = \frac{q^2 V_A^2}{3}\frac{k_z^2 A}{m_0 \gamma_\nu}\cdot \quad (50)$$

For the non-degenerate roots $\omega \simeq k_z V_\nu - qc_\nu + \mathrm{i}\delta$ and $\omega = \omega_- + \mathrm{i}\delta$, we obtain the growth rate

$$\delta \sim \frac{1}{2}\left(\sqrt{\frac{2}{3}}\frac{1}{\cos\Theta}\frac{1}{c^2}\frac{A}{m_0}\right)^{\frac{1}{2}} k_z V_A, \quad (51)$$

which takes the form

$$\delta \sim (10^{-35}B_0)^{1/2}k_z V_A. \quad (52)$$

Next, we consider non-relativistic Alfvén waves for which $V_A^2 \ll c^2$. Introducing $\Omega = \omega - k_z u_0$, we can rewrite equation (44) as

$$\left(\Omega^2 - \omega_-^2\right)\left(\Omega^2 - \omega_+^2\right)\frac{\delta n}{n_0} = \frac{\varepsilon_\nu c^2}{m_0 \gamma_0 V_A^2} \times \left[k_\perp^2 \Omega^2 + \frac{3k_z^2}{(2\gamma_0^2 + 1)}\left(\Omega^2 - k^2 V_A^2\right)\right]\frac{\delta n_\nu}{n_{0\nu}}, \quad (53)$$

where

$$\omega_\mp^2 = [(k^2V_A^2 + k^2c_\nu^2)/2] \mp k^2\sqrt{(V_A^2 + c_\nu^2)^2 - 4V_A^2 c_\nu^2 \cos^2\Theta}$$

are the fast and slow magnetosonic waves. For the magnetic field perturbations, we have

$$\left(\Omega^2 - k^2 V_A^2\right)\frac{\delta B_z}{B_0} = \frac{k_\perp^2 c^2}{(2\gamma_0^2 + 1)}\frac{\delta n}{n_0} + \frac{k_\perp^2 \varepsilon_\nu}{m_0 \gamma_0}\frac{\delta n_\nu}{n_{0\nu}}\cdot \quad (54)$$

We suppose that neutrinos as well as electron-positrons are strongly relativistic and $\gamma_0 \sim \gamma_{0\nu}$. In this case, equation (33) can be written as

$$\left(\Omega^2 - q^2 c_\nu^2\right)\frac{\delta n_\nu}{n_{0\nu}} = q^2 \frac{\delta U_{ep}}{m_{0\nu}\gamma_0}\cdot \quad (55)$$

Having equations (53–55) and the expression (34) for the fast magnetosonic waves ($\Omega^2 \sim \omega_+^2$, $\Omega^2 \gg \omega_-^2$), we obtain the dispersion relation as

$$\left(\Omega^2 - q^2 c_\nu^2\right)\left(\Omega^2 - \omega_+^2\right) = \frac{q^4 \gamma_0}{(2\gamma_0^2 + 1)}\frac{Ac^2}{m_0}\cdot \quad (56)$$

Again for $\Omega \simeq qc_\nu + \mathrm{i}\delta$ and $\omega_+ + \mathrm{i}\delta$, we obtain

$$\delta^2 = \frac{q^3 \gamma_0}{4(2\gamma_0^2 + 1)c_\nu \omega_+}\frac{Ac^2}{m_0}\cdot \quad (57)$$

For the slow magneto-sound waves we have

$$\frac{\delta n_e}{n_0} = \frac{k^2 V_A^2}{\omega_+^2}\frac{3k_z^2}{(2\gamma_0^2 + 1)}\frac{\varepsilon_\nu}{m_0 \gamma_0}\frac{1}{(\Omega^2 - \omega_-^2)}\frac{\delta n_\nu}{n_{0\nu}}, \quad (58)$$

or for the coinciding roots $\Omega \simeq \omega_- + \Delta \approx qc_\nu$, we have

$$\frac{\delta n_e}{n_0} = \frac{k^2 V_A^2}{2\omega_+^2}\frac{3k_z^2}{(2\gamma_0^2 + 1)}\frac{\varepsilon_\nu}{m_0 \gamma_0}\frac{1}{\omega_- \Delta}\frac{\delta n_\nu}{n_{0\nu}}\cdot \quad (59)$$

For the slow magneto-sound waves, excited by the neutrino flux, the growth rate is

$$\delta = \omega_-\left(\frac{3}{4}\frac{k^2 V_A^2}{\omega_+^2}\frac{qk_z^2}{\omega_-^3}\frac{Ac}{m_0}\right)^{\frac{1}{2}}. \quad (60)$$

The above analyses reveal that intense neutrino beams efficiently generate MHD waves in a very dense, strongly magnetized pair plasma.

6 Discussion and conclusions

In this paper, we have developed the electrodynamics of nonlinearly interacting intense neutrino bursts and dense pair plasmas in an external magnetic field. Accounting for the fully relativistic effects, we have derived the governing equations for the neutrinos and a set of RMHD equations for pairs including the neutrino driving force. The governing equations are Fourier analyzed to obtain dispersion relations for MHD waves in the presence of intense neutrino beams. The dispersion relations are analytically analyzed for several interesting cases. It is found that the free energy of the neutrino beams is coupled to Alfvén-like waves which arise in a relativistic pair plasma. For typical supernova plasma parameters, the growth rates of the MHD waves are quite substantial. The neutrino driven MHD waves constitute a new turbulent state in supernova as well as in the magnetosphere of neutron stars. Thus, the frequency spectra can provide valuable information regarding the ambient plasma parameters in astrophysical settings. On the other hand, parametrically excited large amplitude MHD waves are also expected to play a role in accelerating particles and causing non-thermal transport in a very dense magnetoplasma that contains relativistic neutrino beams and electron-positron pairs. In conclusion, it is possible that collective plasma interactions, as described here, may help to understand the origin of gamma-ray bursts that are produced by ultra-relativistic electron-positron plasma jets (fire balls) in a compact region where a vast reservoir of neutrinos, with energy $\geq 10^{51}$ erg, exists.

This research was partially supported by the European Commission (Brussels) through the project (INTAS-GEORGIA-97-052) entitled “Superstrong electromagnetic waves in plasmas”, the Deutsche Forschungsgemeinschaft and the Swedish Research Council through the contract No. 629-2001-2274.

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